Models of Baryogenesis via Spontaneous Lorentz Violation

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Abstract

In the presence of background fields that spontaneously violate Lorentz invariance, a matter-antimatter asymmetry can be generated even in thermal equilibrium. In this paper we systematically investigate models of this type, showing that either high-energy or electroweak versions of baryogenesis are possible, depending on the dynamics of the Lorentz-violating fields. In addition to the previously-studied models of spontaneous baryogenesis and quintessential baryogenesis, we identify two scenarios of interest: baryogenesis from a weak-scale pseudo-Nambu-Goldstone boson with intermediate-scale baryon-number violation, and sphaleron-induced baryogenesis driven by a constant-magnitude vector with a late-time phase transition.

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1 Introduction

The observed universe manifests a pronounced asymmetry between the number density of baryons n_b and antibaryons $n_{\bar{b}}$ (see *e.g.* [1]). Numerically, the baryon-to-entropy ratio is

$$\eta_B \equiv \frac{n_B}{n_\gamma} = 9.2^{+0.6}_{-0.4} \times 10^{-11}\,, \tag{1}$$

where $n_B = n_b - n_{\bar{b}}$ and n_γ are the baryon and photon number density, respectively. However, the origin of the baryon number asymmetry remains a major puzzle for cosmology and particle physics.

In a classic work, Sakharov argued that three conditions are necessary to dynamically generate a baryon asymmetry in an initially baryon-symmetric universe: (1) baryon number non-conserving interactions; (2) C and CP violation; (3) departure from thermal equilibrium [2, 3]. In deriving these conditions, however, the assumption is made that CPT is conserved, as it will always be in a Lorentz-invariant local quantum field theory. If Lorentz invariance is violated, CPT may also be violated, and the assumption of departure from thermal equilibrium is no longer necessary [4, 5, 6, 7]. A concrete implementation of this idea is given by the "spontaneous baryogenesis" [8] scenario, which has subsequently been elaborated upon in various ways [5, 9, 10, 11, 12, 13, 14, 15].

Lorentz invariance is spontaneously violated whenever a tensor field has a nonzero expectation value. In recent years there has been considerable interest in this possibility in the context of various quantum field theories, extra dimensions and brane-world scenarios as well as modified gravity and string theories [16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30]. The existence of Lorentz violation leads to interesting implications for neutrino experiments [31], high energy cosmic ray phenomena [16, 32, 33], evolution of the fine-structure constant [34, 35] and Newton's constant [36], the cosmological constant problem [37, 38], dark energy [26, 35, 39], inflation [40], and the cosmic microwave background (CMB) [41].

In this paper we consider a vector field A_μ with a timelike expectation value in a cosmological background. The norm of the vector need not be constant, nor need the vector field be fundamental; we consider different possible dynamical origins for the field, including the possibility that it is the gradient of a scalar. A coupling to the baryon-number current J^μ_B of the form

$$\mathcal{L} = gA_\mu J^\mu_B \tag{2}$$

leads to an effective chemical potential for baryons of the form $\mu = -gA_0$. In such a background, baryons and antibaryons have different masses, and a nonzero value of n_B will arise even in thermal equilibrium. Depending on the behavior of μ/T, where T is the temperature of the cosmological plasma, a relic baryon asymmetry can be generated by interactions that violate either $B+L$ or $B-L$, as we explore below.

We identify two scenarios of potential interest. One is the case of a simple constant-magnitude timelike vector field coupled to J^μ_{B+L}. In this case we calculate that there can be an appropriate baryon asymmetry generated by electroweak sphalerons *alone*. However, the required magnitude for the vector is in conflict with bounds from present-day experiments, so it is necessary to invoke a late-time phase transition to eliminate

the vector today. The other possibility is that of a derivatively-coupled pseudo-Nambu-Goldstone boson. We find that the PNGB mass parameter required to generate the correct asymmetry is naturally at the weak scale. This scenario requires $(B-L)$-violating interactions that freeze out at an intermediate scale of order 10^{10} GeV, which is perfectly reasonable in models of Majorana neutrino masses. We believe that this model is worthy of further study.

The outline of this paper is as follows. The next section describes how a baryon asymmetry will arise in the presence of spontaneous Lorentz violation. We pay particular attention to the calculation of the relevant freeze-out temperature, which depends on the sources of baryon-number violation as well as the dynamics of the background vector field, and we discuss some specific sources of baryon-number violation. A case of special interest is the effect of sphalerons, which often tend to dilute any existing baryon asymmetry; in Section 3 we show that sphalerons can actually be responsible for the observed baryon asymmetry in the presence of an appropriate Lorentz-violating background. In Section 4 we consider some indirect and direct experimental constraints on the vector field A_μ at late times. These bounds are much lower than the required value to generate the right net baryon number density, implying that the vector must decay appreciably between the early universe and today. We then consider models for the Lorentz-violating fields themselves, including both models with a constant vector field (either fundamental or a ghost condensate), and models featuring scalar fields rolling down a potential.

2 Baryogenesis in the presence of Lorentz violation

2.1 Basic mechanism

We consider the theory of a vector field A_μ with a nonzero vacuum expectation value (vev), coupled to a current J^μ. The action is given by

$$\mathcal{S} = \int d^4x\sqrt{-g}(\mathcal{L}_A + \mathcal{L}_{int} + \mathcal{L}_m)\,, \tag{3}$$

where $\mathcal{L}_A$ is the Lagrange density of the vector field, $\mathcal{L}_m$ denotes the Lagrange density for the other matter fields and $\mathcal{L}_{int}$ is the Lagrange density for the interaction term. The vector A_μ is not necessarily the field that is varied to get the equations of motion from this action; it could be the derivative of a scalar field $\partial_\mu\phi$ [8, 9, 10, 11, 12, 13, 26], the vector current of some hidden fermions $\bar{\psi}\gamma^\mu\psi$ [42, 21, 22, 23, 24], the four-divergence of some higher-rank tensor background field $\nabla_\mu\nabla_\nu\ldots T^{\mu\nu\cdots}$ [5, 16], or even the derivative of the scalar curvature $\partial_\mu\mathcal{R}$ [15].

Our concern will be with the effect of the vev for A_μ, so we will not investigate possible forms of the vector kinetic term until Section 5. The crucial feature will be the existence of a timelike expectation value,

$$\langle A_\mu A^\mu\rangle < 0\,. \tag{4}$$

In a flat Robertson-Walker universe with metric

$$ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2]\,, \tag{5}$$

we will take the vector to point purely in the timelike direction, with components

$$A_\mu = (a_0, 0, 0, 0)\ . \tag{6}$$

Note that a_0 is *not* assumed to be constant. Condensation of the vector field clearly leads to spontaneous breaking of Lorentz invariance; we are assuming that the rest frame of the vector field is that of the cosmological fluid. Such a vector field will, of course, have an energy-momentum tensor with a corresponding effect on the expansion rate; for the purposes of this paper, however, we will assume that this energy remains negligible compared to the total energy in the radiation-dominated era.

We take the interaction Lagrangian density to be the natural one[1]

$$\mathcal{L}_{int} = gA_\mu J^\mu\,, \tag{7}$$

where g is a coupling constant and J^μ is the current corresponding to some continuous global symmetry of the matter fields such as baryon number. This current takes the form

$$J^\mu = \sum_i \beta_i \bar{\psi}_i \gamma^\mu \psi_i \tag{8}$$

for some set of dimensionless parameters β_i. For the configuration (6), the interaction becomes

$$\mathcal{L}_{int} = -ga_0 Q\,, \tag{9}$$

where Q is the conserved charge density.

For purposes of illustration, consider the case when J^μ is precisely the baryon number current. Eq. (9) then becomes

$$\mathcal{L}_{int} = -ga_0(n_b - n_{\bar{b}})\,, \tag{10}$$

where n_b is the number density of baryons and $n_{\bar{b}}$ that of antibaryons. The vector-field background spontaneously violates CPT invariance, splitting the masses of the baryons and antibaryons. In the Appendix we verify that this has the effect of giving baryons a chemical potential

$$\mu_b^0 = ga_0\,, \tag{11}$$

with antibaryons getting a corresponding effective chemical potential $\mu_{\bar{b}}^0 = -ga_0$. Because of this effect, in thermal equilibrium there will be a non-zero baryon number density generated by baryon-number violating interactions:

$$n_B = n_b - n_{\bar{b}} = \frac{g_b T^3}{6\pi^2}\Big[\pi^2 \frac{\mu_b^0}{T} + \Big(\frac{\mu_b^0}{T}\Big)^3\Big] \simeq \frac{g_b \mu_b^0 T^2}{6} \sim \mu_b^0 T^2\,, \tag{12}$$

[1]The interaction term here can not be eliminated by a field redefinition because the conservation of the current J_μ is violated by the matter fields, such violation (for example, baryon current violation) is essential to generate the matter-antimatter asymmetry in the universe.

where g_b counts the internal degrees of freedom of the baryons. In thermal equilibrium, the entropy density is $s = (2\pi^2/45)g_{*s}T^3$. When the B-violating interactions mentioned above become ineffective ($\Gamma \leq H$), we get the final baryon asymmetry

$$\frac{n_B}{s} \sim \frac{\mu_b^0}{g_{*s}T_F} = \frac{ga_0}{g_{*s}T_F}\,, \tag{13}$$

where T_F is the temperature at which the baryon number production is frozen out.

This is the effect that we will explore in this paper: the generation of an *equilibrium* baryon asymmetry due to spontaneous Lorentz violation. This phenomenon has been considered previously in various specific contexts [8, 5, 10, 11, 12, 15]; here we consider the scenario in some generality. Our concern is therefore to determine what happens for different sources of baryon-number violation (and hence freeze-out temperatures) and Lorentz violation (and hence chemical potentials).

2.2 Freeze-out temperature

Our first step is to consider possible values of the freeze-out temperature T_F for different sources of baryon-number violation. We know that sphaleron transitions [43] connect baryon number and lepton number, so we need to consider both the baryon number current and lepton number current that couple to the background fields[2]. The interaction Lagrangian will actually be a sum of couplings to the baryon and lepton currents,

$$\mathcal{L}_{int} = g_B A_\mu J_B^\mu + g_L A_\mu J_L^\mu\,. \tag{14}$$

It is convenient to rewrite this interaction in terms of the $B+L$ and $B-L$ currents,

$$\mathcal{L}_{int} = g_- A_\mu J_{B-L}^\mu + g_+ A_\mu J_{B+L}^\mu\,, \tag{15}$$

where

$$J_{B-L} = J_B - J_L, \quad J_{B+L} = J_B + J_L, \tag{16}$$

and

$$g_- = \frac{1}{2}(g_B - g_L), \quad g_+ = \frac{1}{2}(g_B + g_L). \tag{17}$$

Sphaleron transitions will induce a nonzero baryon number density if the Lorentz-violating background is coupled to the J_{B+L}^μ current; that is, if g_+ is nonvanishing. From Eq. (13), we know that

$$\frac{n_{B-L}}{s} = \frac{\mu_-^0(T_-)}{g_{*s}T_-} = \frac{g_- a_0(T_-)}{g_{*s}T_-}\,, \qquad \frac{n_{B+L}}{s} = \frac{\mu_+^0(T_+)}{g_{*s}T_+} = \frac{g_+ a_0(T_+)}{g_{*s}T_+}\,, \tag{18}$$

[2]More generically, not only the lepton current, but any currents derivatively coupled to the scalar field that are non-orthogonal to the baryon number current should be considered, as their relaxation energetically favors a non-zero baryon charge. This is similar to the case of spontaneous electroweak baryogenesis [9].

where T_- and T_+ are the lowest freeze-out temperature for any interactions that could violate $B-L$ and $B+L$, respectively. The ratio n_{B+L}/s arises from sphaleron transitions, so we know that T_+ must be 150 GeV, which is the critical temperature of electroweak phase transition when sphalerons freeze out. If there is no additional symmetry to set $g_- \gg g_+$ or $g_- \gg g_+$, we will assume that g_- and g_+ are of the same order. We also presume that $T_+ = 150$ GeV $\ll T_-$, which is at least in the order of TeV, so whether $n_{B+L} \ll n_{B-L}$ or $n_{B+L} \gg n_{B-L}$ will only depend on whether $\mu^0(T)/T$ is an increasing or decreasing function with respect to $1/T$. (We think of $\mu^0(T)/T$ as a function of $1/T$, rather than T, since the former is evolves monotonically with the scale factor a.) We therefore have

$$\begin{aligned} n_{B+L} \gg n_{B-L} \quad & \text{if } \frac{|\mu^0(T)|}{T} \text{ increases as a function of } 1/T, \\ n_{B+L} \ll n_{B-L} \quad & \text{if } \frac{|\mu^0(T)|}{T} \text{ decreases as a function of } 1/T. \end{aligned} \tag{19}$$

The net baryon number $n_B = (n_{B+L} + n_{B-L})/2$, so we know that n_B is of the same order as $\max\{n_{B+L}, n_{B-L}\}$. From Eq. (19), we get

$$\frac{n_B}{s} = \begin{cases} \dfrac{n_{B+L}}{2s} \sim \dfrac{g_+ a_0(T_+)}{g_{*s} T_+} & \text{if } \dfrac{|\mu^0(T)|}{T} \text{ increases as a function of } 1/T, \\ \dfrac{n_{B-L}}{2s} \sim \dfrac{g_- a_0(T_-)}{g_{*s} T_-} & \text{if } \dfrac{|\mu^0(T)|}{T} \text{ decreases as a function of } 1/T. \end{cases} \tag{20}$$

Thus, the behavior of the effective chemical potential μ^0 as a function of temperature controls the final baryon asymmetry by determining whether the freeze-out temperature T_F is T_+ or T_-. Figure 1 illustrates the two basic possibilities.

The interesting thing in our paper is the former case. That is, if $a_0(T)$ is a constant or even increases with time (or decreases more slowly than T), we will get $n_{B+L}/s \gg n_{B-L}/s$, so that n_B will only depend on the sphaleron freeze-out temperature $T_+ = 150$ GeV. Unfortunately, the corresponding value for μ^0 is naively in conflict with present-day experimental limits, as we discuss in Section 4. In Section 5, we consider various explicit scenarios for the origin of the Lorentz-violating field, and mechanisms by which it may evade experimental constraints.

The latter case, where $a_0(T)$ decreases faster than T, will apply if the interaction arises from higher-power derivatives in the current or a slowly-rolling field. The net baryon number generated from sphaleron transitions may then be neglected if there is any higher-temperature $B-L$ violation, since $n_{B+L}/s \ll n_{B-L}/s$ and n_B will only depend on T_-.

In a nonsupersymmetric model, the most natural way to violate $B-L$ is to introduce a Majorana mass term which violates L by two units. Following the current mass boundary of light left-handed neutrinos from the atmospheric and solar neutrino oscillation experiments [44, 45] and the analysis of WMAP [46] and SDSS [47], the freeze-out temperature T_- is 10^{13} GeV for light neutrinos with hierarchical masses and 10^{11} GeV for light neutrinos with degenerate masses. These limits arise from considering scattering process mediated by heavy right-handed neutrinos [11]. In a supersymmetric model, the

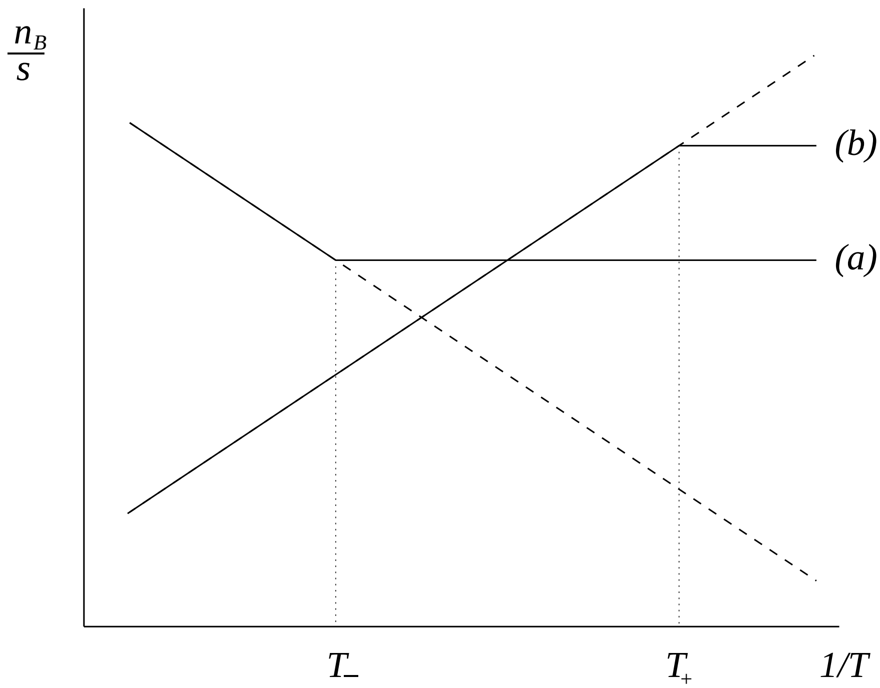

Figure 1: Two cases for the evolution of the baryon-to-entropy ratio in the presence of a Lorentz-violating effective chemical potential μ^0. In case (a), μ^0/T increases or remains constant as a function of $1/T$ (that is, as the universe expands), and the final baryon asymmetry is determined by the temperature T_+ at which $(B+L)$-violating interactions freeze out. Since electroweak sphalerons violate $(B+L)$, this will be at $T_+ \approx 150$ GeV. In case (b), μ^0/T decreases as a function of $1/T$, and the final baryon asymmetry is determined by the temperature T_- at which $(B-L)$-violating interactions freeze out, presumably at some higher temperature.

L-violating interactions (and therefore the $B-L$ violation interactions) are not only from the Majorana mass term, but also from the mass of its scalar partners. We consider the case in which the right-handed neutrinos and their SUSY partner sneutrinos have the mass of order TeV [48, 49]. Then we obtain $T_{-} \approx 1$ TeV since the decay processes will freeze out at a temperature near their mass scale. Note that low-energy baryogenesis offers a way out of the gravitino problem of supersymmetric inflation, since the reheat temperature can be low enough not to overproduce the gravitinos.

3 Baryogenesis via sphalerons

Baryon-number violation in the Standard Model arises from sphaleron transitions. In this section we consider carefully the baryon asymmetry generated by sphalerons in the presence of the effective chemical potential μ induced by Lorentz violation. Our analysis closely follows that of [5], with one important exception: we keep the effective chemical potential induced by (7) in the expression for the free energy, as derived in the Appendix. This opens the possibility of generating the baryon asymmetry from sphalerons without any additional high-energy baryon violation.

Consider N generations of quarks with mass m_{q_i} and leptons with mass m_{l_i}, for $i = 1, \ldots, N$. The free energy in a unit volume for the system in equilibrium at temperature T is given by

$$\mathcal{F} = 6 \sum_{i=1}^{2N} F(m_{q_i}, \mu) + \sum_{i=1}^{N} [2F(m_{l_i}, \mu_i) + F(0, \mu_i)] \,, \tag{21}$$

where the parameters μ and μ_i are the chemical potentials of the quarks and the ith lepton, respectively. We do *not* include the Lorentz-violating contribution μ^0 in the chemical potential, but instead include it explicitly in the expression below for the particle energy. The chemical potentials for the leptons in the same doublet will be equal, since SU(2) interactions are in equilibrium. We neglect the neutrino masses as they are too small to have an important effect. We also do not take the right-handed neutrinos into account in the free energy density; either the right-handed neutrinos have decayed out of equilibrium by the time sphalerons enter into thermal equilibrium, or the right-handed neutrino is simply "sterile" so that it is decoupled from the electroweak anomaly.

In the expression (21), the free energy density for a fermion of mass m and chemical potential μ is given by

$$F(m, \mu) = -T \int \frac{d^3K}{(2\pi)^3} [\ln(1 + e^{-(E+\mu)/T}) + \ln(1 + e^{-(\bar{E}-\mu)/T})] \,, \tag{22}$$

where K_i is the momentum of the fermion, $E = \sqrt{K_i^2 + m^2} + \mu^0$ is the energy of the fermion and $\bar{E} = \sqrt{K_i^2 + m^2} - \mu^0$ is the energy of antifermion.[3] Here we choose to

[3]For the general case, if we need to consider the isocurvature effect of the vector background (see Ref. [50] for such effect in spontaneous baryogenesis), we cannot find a preferred rest frame so that the vector

treat the Lorentz-violating interaction μ^0 as a contribution to the energy of each particle (opposite for baryons and antibaryons), rather than as a contribution to the chemical potential; the two choices are completely equivalent. In Ref. [5], the the additional contribution μ^0 in the fermion energy was neglected; in that case, sphalerons can dilute the baryon density but not produce it.

For quarks and each species of leptons, the μ^0 term is different due to the different coupling strength of the interaction between vector background and the corresponding fermi current. The existence of such a constant energy difference simply shifts the energy of the fermion and antifermion. We use μ^0 and μ_i^0 to denote such energy shift for quarks and the ith species of leptons respectively. The expression for leptonic and baryonic number densities are

$$l_i = \frac{d}{d\mu_i}[2F(m_i, \mu_i) + F(0, \mu_i)] \tag{23}$$

and

$$B = 2\frac{d}{d\mu}\sum_{i=1}^{2N}[F(m_{q_i}, \mu)]\,. \tag{24}$$

(Note that B and n_b are the same quantity.) In the high-temperature approximation $m^2/T^2 \ll 1$, we get

$$F(m, \mu) \approx F(m, 0) - \frac{1}{12}(\mu + \mu^0)^2 T^2 (1 - \frac{3}{2\pi^2}\frac{m^2}{T^2})\,. \tag{25}$$

Sphaleron transitions violate l_i and B, but preserve the combinations

$$L_i = l_i - N^{-1}B\,. \tag{26}$$

The conserved number densities L_i are then given by

$$L_i \equiv l_i - N^{-1}B \approx \frac{(\mu + \mu^0)T^2}{3N}\alpha - \frac{(\mu_i + \mu_i^0)T^2}{2}\beta_i\,, \tag{27}$$

where

$$\alpha \equiv 2N - \frac{3}{2\pi^2}\sum_{i=1}^{2N}\frac{m_{q_i}^2}{T^2}, \qquad \beta_i \equiv 1 - \frac{1}{\pi^2}\frac{m_{l_i}^2}{T^2}\,. \tag{28}$$

Effectively, the sphaleron transitions convert $3N$ quarks and 1 lepton into nothing. In thermal equilibrium, this leads to the relation $\mu = -\sum_i^N \mu_i/3N$. We solve μ_i from Eq.(27) and summing over i leads to the expression

$$\mu = \Big(\frac{2}{T^2}\sum_{i=1}^{N}\frac{L_i}{\beta_i} + \mu_\Delta^0\Big)\Big(\frac{2}{3N}\sum_{i=1}^{N}\frac{\alpha}{\beta_i} + 3N\Big)^{-1}\,, \tag{29}$$

field is purely timelike everywhere. In the expression of the local free energy $F(\vec{x}, m, \mu)$, the energy of the fermion is $E = \sqrt{(\vec{k} - g\tilde{Q}_f\vec{A})^2 + m^2} + \mu^0$ according to Eq. (74). The reason we use the expression $E = \sqrt{(\vec{k} - g\tilde{Q}_f\vec{A})^2 + m^2} + \mu^0$ instead of $E = \sqrt{k^2 + m^2} + \mu^0$ is that we integrate over the canonical conjugate momentum $\vec{P}$ in the phase space of the microcanonical ensemble.

where

$$\mu_\Delta^0 = \sum_{i=1}^{N} \mu_i^0 - \frac{2\alpha}{3N}\mu^0 \sum_{i=1}^{N} \frac{1}{\beta_i} \,. \tag{30}$$

From the baryonic density Eq. (24), we get

$$B = -\frac{1}{3}(\mu + \mu^0)T^2\alpha \,. \tag{31}$$

Plugging in μ, the final baryon number density can be written

$$B = B^{(0)} + B^{(\mu)} \,, \tag{32}$$

with

$$B^{(0)} = -2\alpha\Big(\sum_{i=1}^{N} \frac{L_i}{\beta_i}\Big)\Big(9N + \frac{2\alpha}{N}\sum_{j=1}^{N}\frac{1}{\beta_i}\Big)^{-1} \tag{33}$$

and

$$B^{(\mu)} = \frac{1}{3}\alpha T^2\Big[\mu^0 + 3\mu_\Delta^0\Big(\frac{2\alpha}{N}\sum_{j=1}^{N}\frac{1}{\beta_j} + 9N\Big)^{-1}\Big] \,. \tag{34}$$

$B^{(0)}$ is the conventional baryon number in the presence of sphalerons, which can be expressed as

$$B^{(0)} = \begin{cases} -\dfrac{4}{13\pi^2}\displaystyle\sum_{i=1}^{N} L_i \frac{m_{l_i}^2}{T^2} & B - L = 0 \\ \frac{4}{13}(B-L) & B - L \neq 0 \,. \end{cases} \tag{35}$$

This is the result obtained in Ref. [51]. If the initial $B - L$ is nonzero (usually a nonzero L) when sphalerons enter into thermal equilibrium, then sphaleron transitions will not wash out the initial B asymmetry and will convert an L asymmetry into B asymmetry in some cases. If the initial $B - L$ is zero, taking the leptoquark decay to be dominated by the heaviest lepton τ and the free-out temperature at the electroweak scale, we will get a dilution by a factor of about 10^{-6} [51, 52].

For the term generated by the Lorentz-violating interaction, at leading order $\alpha \sim 2N$, and $\beta_i \sim 1$, and we obtain

$$B^{(\mu)} = -\frac{2N}{13}T^2(3\mu^0 + \frac{1}{N}\sum_{i=1}^{N}\mu_i^0) \,. \tag{36}$$

If there is leptonic flavor violation in thermal equilibrium, we will consider the vector background coupled to the lepton current directly, so the sum over all the species of the effective chemical potential will become a single one $\mu_L^0 \equiv \dfrac{1}{N}\displaystyle\sum_{i=1}^{N}\mu_i^0$. Since each quark carries baryon number 1/3, the effective chemical potential for quarks is $\mu_B^0 = 3\mu^0$. So Eq. (35) becomes $B \propto (\mu_B^0 + \mu_L^0) = 2\mu_{B+L}^0$.

We therefore see that the baryon asymmetry in the presence of the interaction (7) is proportional to $\mu^0 T^2$, just as in (12). Thus, a nonzero net baryon number density can in principle be spontaneously generated through sphaleron transitions in thermal equilibrium in the presence of a nonzero time-like vector background coupled to J_{B+L} current.

4 Present-day constraints on Lorentz violation

We have calculated the baryon-to-entropy ratio n_B/s as a function of the coupling g, the vector field magnitude a_0, and the freeze-out temperature T_F, and argued that low-temperature baryogenesis is possible for sufficiently large $\mu^0 = ga_0$. However, we must take into account the experimental constraints on this parameter in the present-day universe. Although the constraints are not airtight, we find that they are in apparent conflict with the values of ga_0 required for low-temperature baryogenesis in the absence of fine-tuning. It is therefore necessary to consider models in which a_0 decays between early times and today.

4.1 Direct Constraints from mesons

At present we do not observe baryon number violation, so the coupling $a_\mu \bar{\psi}\gamma^\mu\psi$ between the baryon number current and the background field can be eliminated by a field redefinition $\psi \to \exp(ia_\mu x^\mu)\psi$. However, for any experiments involving two species interacting nontrivially (for example, the mass mixing between different generation of quarks), the difference Δa_μ between the corresponding two a_μ coefficients is observable. For the quark sector, the mixing of neutral mesons provides an example where this can be studied [53, 54]. The experimental constraint comes from the parameter

$$\Delta a_\mu = r_{q_1} a_\mu^{q_1} - r_{q_2} a_\mu^{q_2} \,, \tag{37}$$

where $a_\mu^{q_1}$, $a_\mu^{q_2}$ are Lorentz-violating coupling constants for the two valence quarks in the meson, and where the factors r_{q_1} and r_{q_2} allow for quark-binding or other normalization effects [53]. Experiments studying neutral K-mesons have achieved sensitivity to two combinations of components of Δa involving the a_μ coefficients for d and s quarks, with bounds in the Sun-centered frame of approximately

$$|\Delta a_0| \leq 10^{-20} \text{ GeV} \tag{38}$$

by the KTeV Collaboration at Fermilab [55, 56]. Other experiments with D mesons have constrained two combinations of Δa for the u and c quarks at about 10^{-15} GeV (FOCUS Collaboration, Fermilab) [55, 57].

Although there exists the possibility of a delicate cancellation between the terms $r_{q_i} a_\mu^{q_i}$ contributing to Δa_μ, such an arrangement requires substantial fine-tuning. As we will see in Section 5, we require a much larger expectation value a_0 than allowed by these constraints. It is therefore necessary to invoke some mechanism by which the value of a_0 changes substantially between the early universe and today.

4.2 Constraints from the axial vector current

We consider the axial vector current for standard model fermions, whose corresponding axial $U(1)$ symmetry is violated by the Dirac mass term $m_D\bar{\psi}^c\psi$.

$$\mathcal{L}_{int} = g\bar{\psi}\gamma^\mu\gamma^5\psi A_\mu \to g a_0 \bar{\psi}\gamma^\mu\gamma^5\psi \,. \tag{39}$$

If the earth is moving with respect to the rest frame, then after a Lorentz boost, it looks like the interaction $g\mu\bar{\psi}\vec{\gamma}\gamma^5\psi \cdot \vec{v}_{earth}$. In the non-relativistic limit, the current $\bar{\psi}\vec{\gamma}\gamma^5\psi$ is identified with the spin density s, giving us a direct coupling between the velocity of the earth and fermion spin $g\mu\vec{s}\cdot\vec{v}_{earth}$.

Experimental limits on such couplings have placed considerable bounds on ga_0. If we assume that the local rest frame is the same as the rest frame of the CMBR, then $|\vec{v}_{earth}| \sim 10^{-3}$. The bound on couplings to electrons is $ga_0 \sim 10^{-25}$ GeV [58] and to nucleons $ga_0 \sim 10^{-24}$ GeV [59, 60]. These bounds put a strong limit on the combination ga_0.

4.3 Astrophysical constraints

If the Lorentz-violating field arises as the gradient of a scalar, $A_\mu = \partial_\mu\phi/f$, the chiral anomaly can induce a coupling between ϕ and electromagnetism of the form

$$\mathcal{L}_{int} = \frac{\gamma}{f}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}\,. \tag{40}$$

where $F^{\mu\nu}$ is the electromagnetic field strength tensor and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is its dual. The dimensionless coupling γ is typically of order $e^2/4\pi^2$, and the dimensionful parameter f sets the scale of variation for ϕ. (It is, however, possible to avoid this term entirely [26].)

A time-varying ϕ field would rotate the direction of polarization of light from distant radio sources [61]. The dispersion relation for electromagnetic radiation in the presence of a time-dependent ϕ becomes $\omega^2 = k^2 \pm (\gamma/f)\dot{\phi}k$, where $+/-$ refer to right- and left-handed circularly polarized modes, respectively. If we define χ to be the angle between some fiducial direction in the plane of the sky and the polarization vector from an astrophysical source, then in the WKB limit where the wavelength of the radiation is much less than that of ϕ, the difference in group velocity for the two modes leads to a rotation $\Delta\chi = \gamma(\Delta\phi)/f$. If we can assume that $\dot{\phi}$ is roughly constant, this becomes $\Delta\chi = \gamma\dot{\phi}(\Delta t)/f$, where $\Delta t = t|_z - t|_{z=0}$.

We use the data collected by Leahy [62], the most stringent bound on parameter a_0 comes from the single source 3C9 at $z = 2.012$, which reads $\Delta\chi = 2^\circ \pm 3^\circ$, and is consistent with the detailed analysis of Ref. [63]. This gives us a tight bound on the parameter

$$a_0 \sim \frac{\dot{\phi}}{f} \leq \frac{\Delta\chi}{\gamma\Delta t} \sim 10^{-42}\text{GeV}\,. \tag{41}$$

Although this constraint is extremely stringent, we take it somewhat less seriously than (38), since the interaction (40) may be set to zero by an appropriate choice of dynamics.

5 Sources of Lorentz violation

We now turn to the origin of the Lorentz-violating vector field A_μ. We first examine the possibility that A_μ has a constant expectation value in the vacuum ($a_0 =$ constant). This

may arise either from fundamental vector (or higher-rank tensor) fields, or as the gradient of a ghost condensate scalar field. We then consider slowly-rolling scalars, for which the gradient $\partial_\mu \phi$ may not be constant. In either case, we need to consider dynamics which allows for the expectation value a_0 to be large enough to generate an appropriate baryon asymmetry through (20), while avoiding constraints such as (38).

5.1 Fundamental vector fields

We consider a simple Lagrangian for a vector field A_μ and a fermion $\psi(x)$. For simplicity we choose the kinetic term for the vector to be that of ordinary electrodynamics, although there is no gauge invariance associated with the vector. Nothing of importance changes if we also include terms of the form $(\nabla_\mu A^\mu)^2$ or $(\nabla_{(\mu} A_{\nu)})^2$; see [41, 64] for discussions of the gravitational effects and positive-energy degrees of freedom associated with such choices.

Along with the kinetic term and coupling to ψ, we introduce a Mexican-hat potential for the vector field, so that it has a nonzero vacuum expectation value. The Lagrange density is thus

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i\gamma\partial + m)\psi - g A_\mu \bar{\psi}\gamma^\mu \psi - \frac{\mu^2}{2} A_\mu A^\mu - \frac{\lambda}{4}(A_\mu A^\mu)^2 \,, \tag{42}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ denotes the field strength tensor for the vector field A_μ. At the classical level, we obtain the minimal ground state energy for a constant timelike vector field $A_\mu = (a_0, 0, 0, 0)$, with the timelike component given by

$$ {a_0}^2 = \frac{\mu^2}{\lambda}. \tag{43}$$

Such vector-field condensation leads to spontaneous breaking of Lorentz symmetry for a non-zero time-like vev.

It is then straightforward to calculate the baryon-to-entropy ratio produced by such a field. In terms of the discussion in Section 2.2, we note that $\mu^0/T = g a_0/T$ is increasing as the universe expands (and T decreases). The relevant freeze-out temperature is thus $T_+ = 150$ GeV, due to sphaleron transitions. From eq. (13) we obtain

$$\frac{n_B}{s} \sim \frac{g a_0}{g_{*s} T_+} \sim g a_0 (10^4 \text{ GeV})^{-1} \,. \tag{44}$$

If we plug in ${a_0}^2 = \mu^2/\lambda$ and assume that both λ and g are of order unity, obtaining the correct baryon density $n_B/s \sim 10^{-10}$ requires

$$\mu \sim 1 \text{ keV}. \tag{45}$$

The required Lorentz-violating effects are thus relatively small. Thus, it is straightforward to obtain the correct baryon asymmetry from a vector-field condensate that is constant throughout the electroweak phase transition. However, such a field would seem to violate the experimental constraints discussed in Section 4, unless there is some delicate cancellation that allows A_μ to couple to the baryon current but avoid all other bounds.

One way to accomodate the experimental limits without fine-tuning the interactions is to imagine a phase transition for the A_μ field itself, which suddenly changes its expectation value at some point after the electroweak scale. Here we present one simple (albeit contrived) example.

Consider the same Lagrangian of equation (42) but replace the coefficient of the mass term μ^2 with $(\mu'^2 - \alpha|\Phi|^2)$, where Φ is the Higgs doublet:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma\partial + m)\psi - gA_\mu\bar{\psi}\gamma^\mu\psi - \frac{(\mu'^2 - \alpha|\Phi|^2)}{2}A_\mu^2 - \frac{\lambda}{4}(A_\mu^2)^2\,, \tag{46}$$

At high temperatures, the Higgs expectation value $\langle\Phi\rangle$ vanishes, and we get a non-zero vacuum expectation value of the vector background field. At late times, $|\Phi|^2 = v^2$. If $\mu'^2 - \alpha v^2$ is negative, we get a zero vev for vector background field. So the Lorentz symmetry is restored. (Note that only the inequality $\mu'^2 - \alpha v^2 < 0$ is required, not a strict equality.)

Another possibility, perhaps a more natural one, is that finite-temperature effects lead to symmetry restoration in the potential for A_μ. In other words, we imagine that the coefficients μ and λ are temperature-dependent, such that the thermal corrections for the mass term are negative and the expectation value $\langle A_\mu\rangle$ vanishes at zero temperature. We have not constructed an explicit model along these lines, but this possibility deserves closer investigation.

We can also explore more generic couplings containing high power of derivatives in the current, as in Ref. [5]. Consider an interaction Lagrangian

$$\mathcal{L}_{int} \supset \frac{g\langle T\rangle}{M^k}\bar{\psi}(\gamma^0)^{k+1}(i\partial_0)^k\psi + h.c.\,, \tag{47}$$

where $\langle T\rangle$ is the vacuum expectation value of a Lorentz tensor $T_{\mu_1\cdots\mu_{k+1}}$ with dimension $k+1$, g is a dimensionless coupling constant, and M is large UV cutoff mass scale. In thermal equilibrium, we replace each time derivative with a factor of the the associated fermion energy. Due to the existence of the high power of derivatives in the current, the chemical potential is then no longer a constant; rather, it is

$$\mu \sim g\langle T\rangle\left(\frac{E}{M}\right)^k\,. \tag{48}$$

We then get the baryon number density

$$\frac{n_B}{s} \sim g\frac{\langle T\rangle T_F^{k-1}}{g_{*s}M^k}\,. \tag{49}$$

If $k \geq 2$, the baryon number density is proportional to a *positive* power of the freeze-out temperature T_F.

We therefore see that it is possible to get a phenomenologically acceptable baryon asymmetry through a vector field with a constant expectation value at temperatures at and above the electroweak scale. Of course, the need for a phase transition to subsequently eliminate this expectation value renders the mechanism significantly less attractive.

5.2 Ghost condensates

We now turn to the possibility that the Lorentz-violating field arises as the gradient of a scalar,

$$A_\mu = \frac{1}{f}\partial_\mu\phi\,, \tag{50}$$

with f some parameter with units of mass. If the field ϕ has an exact shift symmetry $\phi \to \phi + c$, it always appears as derivatives in the action. Let us assume that it has a wrong-sign quadratic kinetic term, i.e., it is a ghost field [26].

$$\mathcal{L} = +\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \cdots\,. \tag{51}$$

In the spirit of k-essence [65], we assume that the Lagrangian density has the general form $\mathcal{L} = P(X)$, where $X = -\partial^\mu\phi\partial_\mu\phi$. The equation of motion is

$$\partial_\mu[P'(X)\partial^\mu\phi] = 0\,. \tag{52}$$

From this we obtain a solution provided that $\partial_\mu\phi$ = constant. In this case, then, the gradient $\partial_\mu\phi$ behaves precisely like the constant vector field A_μ previously considered.

If $\partial_\mu\phi$ is time like, there is a particular Lorentz frame where $\phi = -M^2 t$. If we consider small fluctuations about this ground state, the field can be written

$$\phi = -M^2 t + \pi\,, \tag{53}$$

where the π field is a small fluctuation. The kinetic energy is stable against small excitations of π provided that $P'(M^4) > 0$ and $P'(M^4) + 2M^4 P''(M^4) > 0$ (so that the kinetic energy and spatial gradient terms have the usual signs).

Then let us consider the direct couplings between the ghost field ϕ and the Standard Model fields. The corresponding effective Lagrangian density is

$$\mathcal{L}_{int} = \frac{g}{f}\partial_\mu\phi J^\mu\,. \tag{54}$$

(Note that this coupling violates the $\phi \to -\phi$ symmetry, which is equivalent to violating time-reversal invariance after the ghost field condenses. Note also that the leading interaction of the ghost condensate is typically to axial vector currents, while our mechanism requires coupling to the baryon or lepton vector currents.) After the ghost field condenses so that $\phi \equiv -M^2 t + \pi$, we have

$$\mathcal{L}_{int} = \frac{g}{f}(-M^2 J_0 + J^\mu\partial_\mu\pi)\,. \tag{55}$$

If J^μ is the baryon number current, the first term in Eq. (55) becomes $-\frac{g}{f}M^2(n_b - n_{\bar{b}})$. The second term could be neglected not only because it is small, but also the small fluctuating field π is oscillating around its minimum, which tends to cancel its contributions [66]. Plugging in $a_0 = gM^2/f$ in Eq. (13) we get:

$$n_B/s \sim \frac{gM^2}{f g_{*s} T_F}\,, \tag{56}$$

This scenario is very similar in spirit to that of the fundamental vector field. Once again, in order to avoid experimental bounds it is necessary to have the ghost expectation value diminish dynamically between the early universe and today.

5.3 Quintessential baryogenesis

If the effective chemical potential is time-dependent or, equivalently, temperature-dependent, we will still get a nonzero net baryon density. A simple way to achieve this is to return to the possibility that A_μ is the gradient of a scalar field ϕ, but now imagine that ϕ is a slowly-rolling quintessence field [67, 68] rather than a ghost condensate [10, 11, 12, 15]. The chemical potential term a_0 is then given by $\dot{\phi}/f$ [10, 11, 12]. Quintessence models may be chosen to have "tracking" behavior [69], in which

$$\dot{\phi} \propto \sqrt{V(\phi)} \propto \sqrt{\rho_{\rm back}}\,. \tag{57}$$

In this case, during the radiation-dominated era $\dot{\phi}$ is proportional to T^2. We follow the result of Ref. [11], where the final net baryon density is

$$\frac{n_B}{s} \approx 0.01g\frac{T_F}{f}\,, \tag{58}$$

where f is the cut off scale for the effective coupling between Lorentz-violating backgrounds and the baryon/lepton currents. Since $\mu^0 = -g\dot{\phi}/f \propto T^2$, according to Eq. (19), $T_F = T_-$ if there is $B-L$ violation in the early universe. Another possibility is that there is no $B-L$ violation at all, so T_F must be $T_+ \approx 150$ GeV. In both cases, it is possible that the freeze-out temperature T_F could be around 1 TeV, and we can obtain the right baryon number density with a cut off f at the intermediate energy scale around 10^{10} GeV.

A final model along these lines [15] considers the interaction between the derivative of the Ricci scalar curvature $\mathcal{R}$ and the baryon number current J^μ from the effective theory of gravity:

$$\frac{1}{M_*^2}\int d^4x\sqrt{-g}(\partial_\mu\mathcal{R})J^\mu\,, \tag{59}$$

The net baryon number density obtained is proportional to an even higher positive power of freeze-out temperature T_F, since $\dot{\mathcal{R}} \propto \dot{\rho}$, where ρ is the total energy density.

5.4 Baryogenesis from pseudo-Nambu-Goldstone bosons

If the Lorentz-violating background arises as the gradient of a scalar field, one way to avoid equivalence-principle violating Yukawa couplings is to imagine there is an approximate shift symmetry $\phi \to \phi + \text{constant}$ [70]. The field is then a pseudo-Nambu-Goldstone boson (PNGB), parametrized by a periodic variable $\theta \sim \theta + 2\pi$, which can be thought of as the angular degree of freedom in a tilted Mexican hat potential. Quintessential baryogenesis (and its relatives) imagines a field that evolves gradually throughout the

history of the universe, perhaps with an energy density tracking that of the background. In contrast, a PNGB will remain overdamped in its potential until the mass parameter becomes comparable to the Hubble parameter, at which time it will roll to its minimum and begin to oscillate. During the initial rolling phase, the gradient $\partial_\mu \phi$ acts like a Lorentz-violating vector field, and leads to an intriguing scenario for baryogenesis.

We consider a generic Lagrangian of the form [71]

$$\mathcal{L} = \frac{f^2}{2}(\partial\theta)^2 - \Lambda^4[1 - \cos(\theta)] \,. \tag{60}$$

The canonically normalized field is $\phi = f\theta$, and the PNGB mass is

$$m = \frac{\Lambda^2}{f} \,. \tag{61}$$

Here, f is the scale of spontaneous symmetry breaking giving rise to the Mexican-hat potential, and Λ is the scale of explicit symmetry breaking that tilts the hat. If we want to generate a baryon asymmetry of the right amplitude, then from

$$\frac{n_B}{s} \sim \frac{\dot{\phi}}{f g_{*s} T_F} = 10^{-10} \,, \tag{62}$$

with $g_{*s} \sim 100$ we require

$$\dot{\phi} = 10^{-8} f T_F \,. \tag{63}$$

The PNGB obeys the equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0 \,. \tag{64}$$

If the field is oscillating near its minimum, $\dot{\phi}$ will change sign quickly, cancelling any asymmetry [66]. We are therefore interested in the damped or overdamped regime, in which we can ignore the second derivative term. For typical values $\phi \sim f$, we then have

$$\dot{\phi} \sim H^{-1}\frac{dV}{d\phi} \sim \frac{m^2 f M_{pl}}{T^2} \,, \tag{65}$$

where T is the temperature. Thus, to achieve successful baryogenesis requires that the freeze-out temperature satisfy

$$\frac{T_F^3}{m^2} \sim 10^8 M_{pl} \,. \tag{66}$$

The ideal circumstance would be if freeze-out occured when the field had just begun to roll substantially (becoming critically damped rather than overdamped), but not yet begun to oscillate. This corresponds to $H \sim m$, which implies

$$T_F^2 \sim m M_{pl} \,. \tag{67}$$

Comparing to (66) shows that PNGB baryogenesis works if the freeze-out temperature is an intermediate scale

$$T_F \sim 10^{-8} M_{pl} \sim 10^{10} \text{ GeV} \tag{68}$$

and the PNGB mass is

$$m \sim \frac{T_F^2}{M_{pl}} \sim 100 \text{ GeV} . \tag{69}$$

The latter number is right around the electroweak scale, which is an encouraging value for a moduli mass. Note that the condition $m \sim T_F^2/M_{pl}$ bears a resemblance to the PNGB formula $m = \Lambda^2/f$; our conditions imply a model in which f is near the Planck scale, and the explicit symmetry-breaking scale Λ (perhaps arising from strong dynamics) is at an intermediate scale around 10^{10} GeV. These requirements seem quite reasonable; in particular, PNGB's with spontaneous-symmetry-braking scales $f \sim M_{pl}$ arise naturally as axions in string theory.

This scenario depends on two separate conditions: the PNGB must have a weak-scale mass, *and* the freeze-out temperature must be (68). It is perhaps fair, however, to turn this around with a more optimistic spin: if $(B-L)$-violating interactions (so that the asymmetry is not diluted by sphalerons) freeze out at an intermediate scale around 10^{10} GeV, a rolling PNGB with electroweak-scale mass and a derivative coupling to J^μ_{B-L} will naturally give rise to a baryon asymmetry of the correct magnitude. In fact it is quite natural to imagine $(B-L)$-violating interactions that freeze out around this scale, arising for example from Majorana neutrino masses. This scenario seems to be worthy of further investigation.

6 Discussion

We have investigated the possible origin of the observed baryon asymmetry in the presence of a coupling between a Lorentz-violating vector field and the baryon current. In the right circumstances it is possible to generate the asymmetry without any baryon-violating interaction beyond the Standard Model, as sphaleron transitions will violate baryon number and drive a matter and antimatter asymmetry in thermal equilibrium. Baryon violation that freezes out at the weak scale can also be converted to an appropriate asymmetry by the evolution of pseudo-Nambu-Goldstone bosons with weak-scale masses. The generic conditions and our comments on such kind of baryogenesis are summarized as follows:

- We need a background field with a nonzero time-like vev which spontaneously breaks Lorentz invariance in the early universe. The vev is not necessarily a constant, but can vary with time or even change discontinuously through phase transitions.

- Such background field may couple to the baryon number current J^μ_B or the lepton number current J^μ_L, as sphalerons violate $B+L$ and can turn a lepton asymmetry into a baryon asymmetry. In the absence of any symmetry, the couplings $g_- A_\mu J^\mu_{B-L}$ and $g_+ A_\mu J^\mu_{B+L}$ should be of similar magnitude.

- We have considered baryon/lepton-violating process in thermal equilibrium. If a_0/T is increasing with time, then the final net baryon number density is determined by the freeze-out temperature $T_+ \approx 150$ GeV. For the opposite case, the final net baryon number density is determined by the freeze-out temperature T_- which is model-dependent.

Our discussions and conclusions here differ from the usually baryogenesis mechanism in thermal equilibrium. In most previous works [5, 10, 11, 12, 15], the absolute value of the effective chemical is decreasing with time, so that in order to generate a right net baryon number density, we need a high freeze-out temperature T_- as n_{B-L}/s depends on the positive power of T_-. However, in our paper, the Lorentz-violating background can be a constant vev instead of a slow rolling field, so that sphaleron transitions can be the main source to generate the baryon asymmetry and the energy scale for baryogenesis to happen is low.

Finally, we want to discuss some generic features of the background fields here for a successful baryogenesis. We have seen that in order to obtain the right net baryon number density in the early universe, the coupling times the time component of the background field ga_0 should be not too small. However, such spontaneous Lorentz-violating term is highly constrained by the experiment at present, so we need some dynamical mechanism to decrease the time component value of such background field. If the background field is some vector or tensor field, we may imagine that a phase transition occurs in between freeze-out and today, so as to evade the experimental constraint at present. If the background arises as the derivative of a scalar, two possibilities present themselves: the field could roll gradually in a tracking-type potential with a gradually diminishing value of $\dot{\phi}$, or it could roll only temporarily before reaching the bottom of its potential and oscillating. The former possibility leads to quintessential baryogenesis, while the latter is realized in pseudo-Nambu-Goldstone boson models.

Our investigation of the PNGB scenario reveals that the most natural implementation of this idea requires PNGB's with weak-scale masses (100 GeV) and $(B-L)$-violating interactions that freeze out at an intermediate scale of around 10^{10} GeV. The former condition can be satisfied if the spontaneous symmetry-breaking scale f is of order the Planck scale and the explicit symmetry-breaking scale Λ is at the same intermediate scale, while the latter condition arises naturally from the decay of Majorana neutrinos. We therefore consider this scenario to be quite promising.

At least two extensions of our investigation remain for future work. One is that we have only considered baryon/lepton number violation. As we have mentioned in footnote 2, any currents derivatively coupled to the scalar field that are non-orthogonal to the baryon number current could possibly lead to a non-zero net baryon number. A simple case is that the scalar field derivatively couples to a current associated with global symmetry $U(1)_Q$, and an interaction violates baryon number and $U(1)_Q$ simultaneously [13]. More complicated cases should be considered, especially when the baryon-number violation freezes out around the electroweak phase transition. The other case is to consider baryon number violation freeze-out when scalar ϕ is in the oscillation stage, as in spontaneous baryogenesis.

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Appendix: single-particle Hamiltonian

In this Appendix we verify that the effect of the coupling of fermions to a Lorentz-violating vector field is to induce an effective chemical potential μ^0, by evaluating the Hamiltonian for a point particle in such a background.

We consider a particle with interaction Lagrangian density (7). Suppose we have a system of fermions with positions $x_n(t)$ and net conserved fermion number Q. The current J^μ for a single particle in special relativity is [72]

$$J^\mu \equiv \sum_n Q_n \delta^3(x - x_n(t)) \frac{dx_n^\mu(t)}{dt} \, . \tag{70}$$

Integrating over space, the new interaction is

$$L_{int} = -g A_\mu \tilde{Q} \frac{dx^\mu}{dt} \, , \tag{71}$$

where $\tilde{Q}$ is the conserved charge per particle. The canonical momentum P conjugate to the position coordinate x is given by

$$P_\mu \equiv \frac{\partial L}{\partial x^i} = \frac{\partial L_{matter}}{\partial x^i} + \frac{\partial L_{int}}{\partial x^i} = p_i + g A_i \tilde{Q} \, , \tag{72}$$

where $p_i = \gamma m dx_i/dt$ is the ordinary kinetic momentum from the fermion sector and $i = 1, 2, 3$ runs over the spatial components. The corresponding Hamiltonian is

$$\begin{aligned} H &= p_i \frac{dx^i}{dt} - L_{int} - L_{matter} \\ &= (p_i + g A_i \tilde{Q}) \frac{dx^i}{dt} + g\tilde{Q} A_0 - L_{matter} \\ &= P_i \frac{dx^i}{dt} - L_{matter} + g\tilde{Q} A_0 \, . \end{aligned} \tag{73}$$

If we solve for dx_i/dt in terms of canonical momentum and plug it into the Hamiltonian, we will find the Hamiltonian for relativistic fermions coupled to a vector field. (A similar discussion to the physics here could be found in Chapter 12.1 of Ref. [73].)

$$H = \sqrt{(P_i - g\tilde{Q} A_i)^2 + m^2} + g\tilde{Q} A_0 \, . \tag{74}$$

Note that the above result does not depend on the fact whether the vector background field has gauge invariance or not.

The vector background field picks out a nonzero constant vev at the ground state, $\langle A_\mu \rangle = (a_0, 0, 0, 0)$. The interaction Lagrangian density becomes

$$\mathcal{L}_{int} = -g a_0 \tilde{Q} \,. \tag{75}$$

The Hamiltonian for one particle is

$$H = \sqrt{P_i^2 + m^2} + g\tilde{Q} a_0 = \sqrt{{p_i}^2 + m^2} + g\tilde{Q} a_0 \,. \tag{76}$$

We see that the additional term gQa_0 acts like a "potential" term for the relativistic charged particle. In the statistics of relativistic charged fermions, such potential term $gQa_0 = Ng\tilde{Q}a_0 \propto N$, so it can be considered as a contribution to the chemical potential. We define the "effective chemical potential"

$$\mu^0 \equiv g\tilde{Q} a_0 \ . \tag{77}$$

If we have interactions that violate the charge Q (such as baryon number B), then $\sqrt{{p_i}^2 + m^2}$ will not be a conserved number due to energy conservation. This will lead to an asymmetric distribution of the particles and generate a final non-zero net charge Q_f.

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